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BRST Hamiltonian for Bulk Quantized Gauge Theory

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Abstract

By treating the bulk–quantized Yang–Mills theory as a constrained system we obtain a consistent gauge–fixed BRST hamiltonian in the minimal sector. This provides an independent derivation of the 5–d lagrangian bulk action. The ground state is independent of the (anti)ghosts and is interpreted as the solution of the Fokker–Planck equation, thus establishing a direct connection to the Fokker–Planck hamiltonian. The vacuum state correlators are shown to be in agreement with correlators in lagrangian 5–d formulation. It is verified that the complete propagators remain parabolic in one–loop dimensional regularization.

$\mathcal{A}\mathcal{M}\mathcal{S}$-LaTeX

1 Introduction

The usual formulation [?] of 4–d gauge theory is based on the free (euclidean) lagrangian action

$$S = \frac{1}{4}\int_{\mathcal{M}^4} dx\, F_{\mu\nu}F^{\mu\nu} \tag{1.1}$$

where $F_{\mu\nu} = \partial_{[\mu}A_{\nu]} + [A_\mu, A_\nu]$ is the curvature of an SU(N) connection A. In recently introduced *bulk quantization* [?, ?, ?, ?] (that arose from stochastic quantization approaches based on ideas of Parisi and Wu [?, ?]) one adds an extra fifth nonphysical dimension t to the spacetime 4–manifold $\mathcal{M}^4$. All the fields of the theory are then defined on the extended spacetime

$$\varphi(x),\ \ x \in \mathcal{M}^4 \ \ \longrightarrow\ \ \varphi(t,x),\ \ (t,x) \in \mathbf{R}\times\mathcal{M}^4$$

(This t corresponds to the stochastic evolution parameter or the Monte Carlo iteration time for numerical simulation). The connection $A_\mu dx^\mu$ is extended to include a fifth component $A_5 dt$ and one has $F_{5\mu} = \partial_5 A_\mu - \partial_\mu A_5 + [A_5, A_\mu]$.

A set of ghost fields is introduced with two independent $\mathbf{Z}$–gradings (ghost numbers) gh_s and gh_w corresponding to BRST differentials s and w, which raise the respective ghost numbers by one and satisfy

$$(s+w)^2 = 0 \ \ \Rightarrow\ \ s^2 = 0,\ \ w^2 = 0,\ \ sw = -ws \tag{1.2}$$

The operator w provides a BRST implementation of the 5–d gauge symmetry, analogous to the usual BRST operator [?, ?] (usually called s!) connected with Faddeev–Popov ghosts. Its cohomology $H^0(w)$ defines observables. The operator s acts like a rigid supersymmetry operator and has trivial cohomology. Observables are *not* required to be s–exact. Fields with odd total ghost number $gh \equiv gh_s + gh_w$ anticommute. The action of s and w on the fields is defined as

$$\begin{array}{cccc}
sA_\mu = \Psi_\mu & s\Psi_\mu = 0 & s\bar\Psi_\mu = \Pi_\mu & s\Pi_\mu = 0 \\
sA_5 = \Psi_5 & s\Psi_5 = 0 & s\bar\Psi_5 = \Pi_5 & s\Pi_5 = 0 \\
sc = \Phi & s\Phi = 0 & s\bar\Phi = \bar c & s\bar c = 0 \\
s\lambda = \mu & s\mu = 0 & s\bar\mu = \bar\lambda & s\bar\lambda = 0
\end{array} \tag{1.3}$$

$$\begin{array}{cccc}
wA_\nu = D_\nu\lambda & w\Psi_\nu = -[\lambda,\Psi_\nu] - D_\nu\mu & w\bar\Psi_\nu = -\,[\lambda,\bar\Psi_\nu] & w\Pi_\nu = -\,[\lambda,\Pi_\nu] + [\mu,\bar\Psi_\nu] \\
wA_5 = D_5\lambda & w\Psi_5 = -[\lambda,\Psi_5] - D_5\mu & w\bar\Psi_5 = -\,[\lambda,\bar\Psi_5] & w\Pi_5 = -\,[\lambda,\Pi_5] + [\mu,\bar\Psi_5] \\
wc = -\,[\lambda, c] - \mu & w\Phi = -[\lambda,\Phi] + [\mu, c] & w\bar\Phi = -[\lambda,\bar\Phi] & w\bar c = -[\lambda,\bar c,] + [\mu,\bar\Phi]
\end{array}$$

$$w\lambda = -\tfrac{1}{2}[\lambda,\lambda] \qquad w\mu = -[\lambda,\mu] \qquad\qquad w\bar\mu = -[\lambda,\bar\mu] + \bar\Psi_5 \quad w\bar\lambda = -[\lambda,\bar\lambda] + [\mu,\bar\mu] + \Pi_5$$

Here $D_\mu = \partial_\mu + [A_\mu,\ ]$ and $D_5 = \partial_5 + [A_5,\ ]$ denotes the usual gauge covariant derivative. With some obvious renaming of fields this is the BRST algebra of [?], with a minor *exception*. To make the action of w on the quartet $A_5, \Psi_5, \bar\Psi_5, \Pi_5$ symmetric in form, as it now is, to the action of w on $A_\mu, \Psi_\mu, \bar\Psi_\mu, \Pi_\mu$ we made the field redefinitions

The 5–d action for the theory is s–exact and w–closed

$$wI = 0 \tag{1.4}$$

and is given by

$$\begin{aligned} I &= I_0 + I_{\rm gf} \\ I_0 &\equiv \int d^5x s\Big[\bar\Psi_\mu\big(F^{5\mu} - D_\lambda F^{\lambda\mu} + \Pi^\mu + [\bar\Psi^\mu, c]\big) + \bar\Phi\big(\Psi_5 - a'^{-1}D_\mu\Psi^\mu - (D_5 - a'^{-1}D^2)c\big)\Big] \\ I_{\rm gf} &\equiv \int d^5x\, ws\Big[\bar\mu\,\big(A_5 - a^{-1}\partial\cdot A\big)\Big] \end{aligned} \tag{1.5}$$

where a and a' are positive constant parameters. After expansion, the w–exact piece $I_{\rm gf}$ fixes the gauge for A_μ and Ψ_μ to $A_5 = a^{-1}\partial\cdot A$ and $\Psi_5 = a^{-1}\partial\cdot\Psi$. The theory is well–defined in this gauge and one has convergence of longitudinal modes. From the 4–d point of view this axial type 5–d gauge condition actually corresponds to an infinitesimal *gauge transformation* $\delta A_\mu = D_\mu a^{-1}\partial\cdot A$, so there is no Gribov obstruction associated with gauge fixing (see [?]).

Because all free ghost propagators are retarded, closed ghost loops vanish (except for tadpoles which can be ignored). Since ghost number is conserved, as long as one doesn't compute ghost correlators the effect of integrating out the ghosts is simply to suppress the ghosts in the action which, after integrating out Π_μ as well and rescaling t, yields

We will address here the question of finding the proper hamiltonian corresponding to (1.5). An outline of how we proceed is as follows. We consider just I_0, the gauge non–fixed part of the action, and read off the hamiltonian, which has simple first class constraints. One has a choice of whether or not to include $A_5, \Psi_5, \bar\Psi_5, \Pi_5$ among the canonical variables; the phase space without these variables is called the the *minimal sector*. In the hamiltonian formalism the first class constraints are generators of gauge transformations, and hence of w. To quantize the system one needs a BRST gauge–fixed hamiltonian. According to homological BRST theory, a ghost–antighost pair is introduced for each constraint and used to construct a BRST generator Ω for w, which we choose to do in the minimal sector for reasons outlined below. We then obtain a gauge–fixed hamiltonian $H^{\min} = H_C^{\min} - \{\Omega, K\}$, the gauge being fixed by the second term with K chosen so as to give action $I^{\min}$ (a reduced form of I that results after integrating out non–minimal fields).

We then go on to show that the complete ghost propagators remain retarded in one–loop dimensional regularization. The retarded character of the full ghost propagators allows us to establish an equivalence between the quantum hamiltonian and lagrangian correlation functions. We also argue that the ground state wave function P has trivial ghost dependence, which provides a direct connection to the Fokker–Planck equation

$$-\int d^4x \, \frac{\delta}{\delta A_\mu(x)} \Big[\frac{\delta}{\delta A^\mu(x)} - \frac{\delta S_{\rm YM}}{\delta A^\mu(x)} + a^{-1} D_\mu \partial\cdot A(x) \Big] P(A) = 0 \tag{1.6}$$

2 Constrained hamiltonian

The gauge non–fixed part of the action after expansion is

$$\begin{aligned} I_0 &= I_F + I_\Pi + I_c \\ I_F &= \int d^5x \big[\Pi_\mu (F^{5\mu} - D_\lambda F^{\lambda\mu}) - \bar\Psi_\mu \big(D^{[5}\Psi^{\mu]} - D_\lambda D^{[\lambda}\Psi^{\mu]} - [F^{\mu\nu}, \Psi_\nu] \big) \big] \\ I_\Pi &= \int d^5x \big[\Pi^2 + 2\Pi_\mu [\bar\Psi^\mu, c] + [\bar\Psi^\mu, \bar\Psi_\mu] \Phi \big] \\ I_c &= \int d^5x \, \bar c \big[(\Psi_5 - a'^{-1} D_\mu \Psi^\mu - (D_5 - a'^{-1} D^2) c \big] \\ &+ \int d^5x \, \bar\Phi \big[-(D_5 - a'^{-1} D^2)\Phi - [\Psi_5 - a'^{-1} D_\mu \Psi^\mu, c] + a'^{-1} [\Psi^\mu, 2D_\mu c - \Psi_\mu] \big] \end{aligned} \tag{2.1}$$

The gauge fixing term for future reference is

$$\begin{aligned} I_{\rm gf} = \int d^5x \, \big[& \Pi_5 (A^5 - a^{-1} \partial\cdot A) + \bar\Psi_5 (\Psi^5 - a^{-1} \partial_\mu \Psi^\mu) \\ & - \bar\lambda (\partial_5 - a^{-1} D\cdot\partial) \lambda - \bar\mu \big((\partial_5 - a^{-1} D\cdot\partial)\mu - a^{-1} [\Psi^\mu, \partial_\mu \lambda] \big) \big] \end{aligned} \tag{2.2}$$

First we look at the equations of motion generated by varying I_0 with respect to the fields Ψ_5 and A_5

$$\begin{aligned} 0 &= \frac{\delta I_0}{\delta \Psi_5} = D_\mu \bar\Psi^\mu + [\bar\Phi, c] - \bar c \equiv \varphi_1, \\ 0 &= \frac{\delta I_0}{\delta A_5} = D_\mu \Pi^\mu + [\bar\Psi_\mu, \Psi^\mu] + [\bar c, c] + [\bar\Phi, \Phi] \equiv \varphi_2 \end{aligned} \tag{2.3}$$

and obtain what are called *primary constraints* φ_1 and φ_2. Note that we use the usual convention that all functional derivatives with respect to Grassman fields are *left* derivatives. It is of interest to observe that the constraints satisfy

$$s\varphi_1 = \varphi_2 \tag{2.4}$$

We substitute these constraints into the action I_0, and obtain a reduced form of the action $I_0^{\rm min}$, where all terms linear in Ψ_5 and A_5 have been eliminated by the equations of motion (2.3). In this approach Ψ_5 and A_5 play the role of lagrange multipliers and are *not* canonical variables. This is analogous to the role A_0 plays in enforcing Gauss law $D_j E^j = 0$ in the minimal hamiltonian for electromagnetism or Yang–Mills, where only A_j and E_j are treated as canonical variables. Thus

$$I_0 = \int d^5x \left(\dot{A}_\mu \Pi^\mu + \dot{\Psi}_\mu \bar{\Psi}^\mu + \dot{c}\bar{c} - \dot{\Phi}\bar{\Phi} - \mathcal{H}_C^{\rm min} - \Psi^5\varphi_1 - A^5\varphi_2\right) \tag{2.5}$$

and the reduced form of the action is

$$\begin{aligned} I_0^{\rm min} &= \int d^5x \left(\dot{A}_\mu \Pi^\mu + \dot{\Psi}_\mu \bar{\Psi}^\mu + \dot{c}\bar{c} - \dot{\Phi}\bar{\Phi} - \mathcal{H}_C^{\rm min}\right), \qquad (2.6) \\ -\mathcal{H}_C^{\rm min} &= \Pi^2 + \left(2[\bar{\Psi}_\mu, c] - D_\lambda F^{\lambda\mu}\right)\Pi_\mu + \bar{\Psi}_\mu\left(D_\lambda D^{[\lambda}\Psi^{\mu]} + [F^{\mu\nu}, \Psi_\nu]\right) + [\bar{\Psi}_\mu, \bar{\Psi}^\mu]\Phi \\ &\quad + a'^{-1}\bar{c}(D^2 c - D_\mu \Psi^\mu) + a'^{-1}\bar{\Phi}\left(D^2\Phi + [D_\mu\Psi^\mu, c] + [\Psi^\mu, 2D_\mu c - \Psi_\mu]\right) \qquad (2.7) \end{aligned}$$

One reads off $H_C^{\rm min}$ from I_0 by dropping kinetic terms and setting $\varphi_1 = \varphi_2 = 0$. Since I_0, as given in (1.5), is s–exact it follows that $H_C^{\rm min}$ is also s–exact and can be expressed as

$$H_C^{\rm min} = \int d^4x\, s\left[\bar{\Psi}_\mu\left(\Pi^\mu - D_\lambda F^{\lambda\mu} - [\bar{\Psi}^\mu, c]\right) - a'^{-1}\bar{\Phi}D_\mu\left(\Psi^\mu - D^\mu c\right)\right] \tag{2.8}$$

We now proceed with the analysis of this constrained gauge system which goes according to a standard prescription, as follows. The reader is referred to to [**?**, **?**] for background on constrained systems. The constrained hamiltonian is written

We use $H_C^{\rm min}$ to denote the canonical hamiltonian with the corresponding action $I_0^{\rm min}$. The action I_0 with constraints is then called the *extended action* and $H^{\rm min}$ is termed *extended hamiltonian*. The $\approx$ notation is introduced to represent *weak* equality, that is equality modulo functions that vanish on the constraint surface in phase space described by $\varphi_j = 0$. $H^{\rm min}$ then determines time evolution of all functions F of the fields by

The constraints φ_j must be preserved in time, so we apply (**??**) to φ_j and get $\{H_C^{\min}, \varphi_j\} \approx 0$ which generates no further (what would be termed *secondary*) constraints. Some computation (the Jacobi identity is useful) shows that the constraints φ_m close to generate a Lie algebra (the structure functions are all constant)

For constrained hamiltonian systems a functional F whose bracket with every constraint (including secondary, if they are present) vanishes weakly

Now, for each generator φ_m, the corresponding gauge transformation is given by (**??**), but without sum on m. One has

$$\begin{aligned}
\delta_{\epsilon_2} A^a_\mu &= -(D_\mu \epsilon_2)^a & \delta_{\epsilon_1} A^a_\mu &= 0 \\
\delta_{\epsilon_2} \Psi^a_\mu &= [\epsilon_2, \Psi_\mu]^a & \delta_{\epsilon_1} \Psi^a_\mu &= -(D_\mu \epsilon_1)^a \\
\delta_{\epsilon_2} \bar{\Psi}^a_\mu &= [\epsilon_2, \bar{\Psi}_\mu]^a & \delta_{\epsilon_1} \bar{\Psi}^a_\mu &= 0 \\
\delta_{\epsilon_2} \Pi^a_\mu &= [\epsilon_2, \Pi_\mu]^a & \delta_{\epsilon_1} \Pi^a_\mu &= [\epsilon_1, \bar{\Psi}_\mu]^a \\
\delta_{\epsilon_2} c^a &= [\epsilon_2, c]^a & \delta_{\epsilon_1} c^a &= -\epsilon_1^a \\
\delta_{\epsilon_2} \Phi^a &= [\epsilon_2, \Phi]^a & \delta_{\epsilon_1} \Phi^a &= [\epsilon_1, c]^a \\
\delta_{\epsilon_2} \bar{\Phi}^a &= [\epsilon_2, \bar{\Phi}]^a & \delta_{\epsilon_1} \bar{\Phi}^a &= 0 \\
\delta_{\epsilon_2} \bar{c}^a &= [\epsilon_2, \bar{c}]^a & \delta_{\epsilon_1} \bar{c}^a &= [\epsilon_1, \bar{\Phi}]^a
\end{aligned} \tag{2.9}$$

The full gauge transformations (**??**) of the fields are given by

$$\begin{aligned}
&\delta A_\nu = -D_\nu \epsilon_2 \quad && \delta \Psi_\nu = [\epsilon_2, \Psi_\nu] - D_\nu \epsilon_1 \quad && \delta \bar{\Psi}_\nu = [\epsilon_2, \bar{\Psi}_\nu] \quad && \delta \Pi_\nu = [\epsilon_2, \Pi_\nu] + [\epsilon_1, \bar{\Psi}_\nu] \\
&\delta c = [\epsilon_2, c] - \epsilon_1 && \delta \Phi = [\epsilon_2, \Phi] + [\epsilon_1, c] && \delta \bar{\Phi} = [\epsilon_2, \bar{\Phi}] && \delta \bar{c} = [\epsilon_2, \bar{c},] + [\epsilon_1, \bar{\Phi}]
\end{aligned} \tag{2.10}$$

which coincides with the remnant (after the A_5 quartet is gone) of the w algebra (1.3) when the ϵ parameters are replaced by variables of opposite statistics

As for the lagrange multiplier fields, one has the freedom of assigning to them any gauge transformation properties one sees fit, and we may therefore choose to transform them in such a way as to make the entire action

$$I_0 = I_0^{\min} - \int d^5x \, (\Psi_5 \varphi_1 + A_5 \varphi_2) \tag{2.11}$$

gauge invariant. This can always be arranged even in the most general cases with second class constraints [**?**], and in our case amounts to (not surprisingly) setting

$$\delta \Psi_5 = -D_5 \epsilon_1 + [\epsilon_2, \Psi_5], \qquad \delta A_5 = -D_5 \epsilon_2 \tag{2.12}$$

3 Minimal BRST hamiltonian

The following considerations are direct consequences of standard results of homological BRST theory (we refer to [**?**] for details). The extended phase space is introduced by including in the minimal sector a ghost–antighost conjugate pair for each of the constraints φ_1 and φ_2. Hence we add $\mu, \bar{\mu}$ for φ_1 and $\lambda, \bar{\lambda}$ for φ_2, and as the notation indicates, identify them with the ghost fields in $I_{\rm gf}$. The corresponding kinetic terms $-\dot{\mu}\bar{\mu} + \dot{\lambda}\bar{\lambda}$ are included in the action. Note the minus sign in $\{\mu(x), \bar{\mu}(y)\} = -\delta(x-y)$.

By inspection of (1.3), one easily finds the generator Q for s on the extended phase space

$$s = -\{Q, \ \}, \qquad Q = \int d^4x \left(\Psi_\mu \Pi^\mu + \Phi\bar{c} + \mu\bar{\lambda}\right) \tag{3.1}$$

Therefore we have

Thus Ω is Q–exact which implies $\{Q, \Omega\} = 0$, as expected. We remark that in theories where the constraints do not generate a closed algebra with constant structure functions, the BRST generator may be much more complicated (an infinite series expansion in a ghost degree). So again, we see the attractive simplicity of this 5–d theory.

One needs to construct an appropriate BRST invariant extension of $H_C^{\rm min}$ and then gauge–fix it. But $H_C^{\rm min}$ is already w–invariant so the gauge–fixed BRST hamiltonian corresponding to $I = I_0 + I_{\rm gf}$, as expressed in terms of this minimal set of fields, is then simply given by

$$H^{\rm min} = H_C^{\rm min} + H_{\rm gf}^{\rm min} = -\{Q, X\} - \{\Omega, K\} \tag{3.2}$$

where the *gauge fixing fermion* K, as it is frequently called, is chosen to be

Explicitly one has

$$-\{\Omega, K\} \ = \ a^{-1}\int d^4x \left(\varphi_1 \partial\cdot\Psi - \varphi_2\partial\cdot A - \bar{\mu}D\cdot\partial\mu - \bar{\lambda}D\cdot\partial\lambda - \bar{\mu}[\bar{\Psi}^\mu, \partial_\mu\lambda]\right) \tag{3.3}$$

Thus the BRST hamiltonian in its fully expanded form is

$$\begin{aligned} H^{\rm min} &= -\{Q, \bar{Q}\} \\ &= -\{Q, X\} - \{\Omega, K\} \\ &= \int d^4x \ - \Big(\Pi^2 + \Pi_\mu\big(2[\bar{\Psi}_\mu, c] - D_\lambda F^{\lambda\mu}\big) + \bar{\Psi}_\mu\big(D_\lambda D^{[\lambda}\Psi^{\mu]} + [F^{\mu\nu}, \Psi_\nu]\big) + [\bar{\Psi}_\mu, \bar{\Psi}^\mu]\Phi \end{aligned}$$

$$+\frac{1}{a'}\big[\bar{c}(D^2c - D_\mu\Psi^\mu) + \bar{\Phi}\big(D^2\Phi + [D_\mu\Psi^\mu, c] + [\Psi^\mu, 2D_\mu c - \Psi_\mu]\big)\big]\Big) \tag{3.4}$$

$$+\frac{1}{a}\Big((D_\mu\bar{\Psi}^\mu + [\bar{\Phi}, c] - \bar{c})\partial\cdot\Psi - (D_\mu\Pi^\mu + [\bar{\Psi}_\mu, \Psi^\mu] + [\bar{c}, c] + [\bar{\Phi}, \Phi])\partial\cdot A$$
$$-\bar{\mu}D\cdot\partial\mu - \bar{\lambda}D\cdot\partial\lambda - \bar{\mu}[\bar{\Psi}^\mu, \partial_\mu\lambda]\Big)$$

Our canonical treatment agrees with [?] because one can easily check that after integrating out $\Pi_5, A_5, \bar{\Psi}_5, \Psi_5$ in the lagrangian action I [= (2.1) + (2.2)] one gets precisely

$$I^{\min} = \int d^5x\, \big(\dot{A}_\mu\Pi^\mu + \dot{\Psi}_\mu\bar{\Psi}^\mu + \dot{c}\bar{c} - \dot{\Phi}\bar{\Phi} + \dot{\lambda}\bar{\lambda} - \dot{\mu}\bar{\mu} - \mathcal{H}^{\min}\big) \tag{3.5}$$

So in fact, what we have done here is give a consistent constructive derivation of the (reduced form of) action I based on the canonical analysis of the constrained hamiltonian.

4 Propagators

In this section we study the propagators and show that all complete ghost (and ghost of ghost) propagators stay retarded in one-loop dimensional regularization. This is an important feature of bulk quantization and will be key to establishing the advertised results on correlators ant the ground state in the next section..

Let us then begin by first computing the *free* propagators by inverting the quadratic part of the action $I^{\min}$, which is given by the quadratic form

$$I^{\min}_{(0)} = -\int d^5x\; A^\mu(-\delta_{\mu\nu}\partial_5 + a^{-1}\partial_\mu\partial_\nu + \Box^{\mathrm{tr}}_{\mu\nu})\Pi^\nu - \Pi^2$$
$$+\bar{\Phi}(\partial_5 - a'^{-1}\Box)\Phi + \bar{\lambda}(\partial_5 - a'^{-1}\Box)\lambda + \bar{\mu}(\partial_5 - a'^{-1}\Box)\mu$$
$$+\big(\bar{\Psi}_\mu, \bar{c}\big)\begin{pmatrix} \delta_{\mu\nu}\partial_5 - a^{-1}\partial_\mu\partial_\nu - \Box^{\mathrm{tr}}_{\mu\nu} & 0 \\ (a-a')\partial_\nu/aa' & \partial_5 - a'^{-1}\Box \end{pmatrix}\begin{pmatrix} \Psi_\nu \\ c \end{pmatrix} \tag{4.1}$$

Here

$$\Box^{\mathrm{tr}}_{\mu\nu} = \Box P^{\mathrm{tr}} = \delta_{\mu\nu}\Box - \partial_\mu\partial_\nu \tag{4.2}$$

and P^{tr} and P^{lg} are the usual transverse and longitudinal projectors. They may be defined via their Fourier transforms (denoted by $\hat{\ }$)

$$\hat{P}^{\mathrm{tr}}_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}, \qquad \hat{P}^{\mathrm{lg}}_{\mu\nu}(p) = \hat{P}^{\mathrm{tr}\perp}_{\mu\nu}(p) = \frac{p_\mu p_\nu}{p^2}. \tag{4.3}$$

and provide an orthogonal decomposition $\mathbf{1} = P^{\mathrm{tr}}(\partial) + P^{\mathrm{lg}}(\partial)$.

The single blocks are trivial to invert and give the free momentum space propagators

$$\hat{\mathcal{D}}_{0,\Phi\bar{\Phi}}(p) = \frac{1}{ip_5 + p^2/a'}, \qquad \hat{\mathcal{D}}_{0,\lambda\bar{\lambda}}(p) = \hat{\mathcal{D}}_{0,\mu\bar{\mu}}(p) = \frac{1}{ip_5 + p^2/a} \tag{4.4}$$

The 2×2 block is also straightforward to invert (after using integration by parts to generate a lower left term). The non-vanishing transverse free propagators are then

$$\hat{\mathcal{D}}^{\mathrm{tr}}_{0,A^\mu A^\nu}(p) = \frac{2P^{\mathrm{tr}}_{\mu\nu}}{p_5^2 + (p^2)^2}, \qquad \hat{\mathcal{D}}^{\mathrm{tr}}_{0,A^\mu \Pi^\nu}(p) = -\hat{\mathcal{D}}^{\mathrm{tr}}_{0,\Pi^\mu A^\nu}(p) = \frac{2P^{\mathrm{tr}}_{\mu\nu}}{ip_5 + p^2} \tag{4.5}$$

Using the Hodge decomposition for vector fields

Upon taking the inverse Fourier transform

$$\mathcal{D}_0(t,x) = \frac{1}{(2\pi)^5} \int dp_5\, e^{itp_5} \int d^4p\, e^{ix\cdot p} \hat{\mathcal{D}}_0(p_5, p) \tag{4.6}$$

one sees that all the *free* propagators for the ghosts (and ghosts of ghosts) are retarded, since there is no pole in the upper p_5 half–plane (a and a' are positive) and closing the p_5 contour in the lower half–plane gives $\theta(t)$.

What about the complete propagators $\mathcal{D}$ then? For motivation consider the Green's functions G_0 and G (for $\lambda, \bar{\lambda}$ say), satisfying

$$\begin{aligned} (\partial_t - a^{-1}\partial^\mu\partial_\mu)G_0(t-s; x-y) &= \delta(s,y) \\ (\partial_t - a^{-1}D^\mu\partial_\mu)G(t-s; x, y; A) &= \delta(s,y) \end{aligned} \tag{4.7}$$

Of course $G_0 = \mathcal{D}_0$ is just the free propagator, but $G \neq \mathcal{D} = \langle T\lambda(t,x)\bar{\lambda}(s,y)\rangle_{I^{\min}}$ since the (time ordered) correlator involves integration over $\mathscr{D}A$ as well. Nevertheless, it is instructive to look at properties of G prior to integration. From Duhamel's principle [?] we have the following convolution relation between G and G_0.

$$G(t-s; x, y; A) = G_0(t-s; x-y) + \int_s^t d\tau \int d\xi\, G_0(t-\tau; x-\xi)[A^\mu(\xi), \partial_\mu G(\tau; \xi, A)] \tag{4.8}$$

where we have suppressed indices. From this we see that not only is G automatically retarded as well, but for a sufficiently regular A one would conclude that

$$\lim_{t\searrow s} G(t-s; x, y; A) = \lim_{t\searrow s} G_0(t-s; x-y) = \delta(x-y) \tag{4.9}$$

The significance is that canonical commutation relations are *formally* satisfied if one assumes regularity of A and

$$\lim_{t\searrow s} \mathcal{D}(t-s; x, y) \equiv \lim_{t\searrow s} \langle G(t-s; x, y; A)\rangle = \langle \lim_{t\searrow s} G(t-s; x, y; A)\rangle \tag{4.10}$$